

## Gauss–Newton Inversion Algorithm to Estimate Resistivity Parameter Using a Dual-Optimal Grid Approach

Agah D. Garnadi<sup>1</sup> and Hendra Grandis<sup>2</sup>

<sup>1</sup> Dept. Mathematics, Faculty of Mathematics and Natural Sciences, Institut Pertanian Bogor,  
Jl Meranti Kampus Darmaga, Bogor 16680, Indonesia. E-mail: agah.garnadi@gmail.com

<sup>2</sup> Dept. Geophysics, Faculty of Mining and Petroleum Engineering, Institut Teknologi Bandung,  
Jl. Ganesha 10, Bandung 40132, Indonesia. E-mail: grandis@earthling.net

---

### SUMMARY

RESINVM3D is an open source 3-D, MATLAB based, resistivity inversion package. The inversion algorithm employs the Gauss-Newton method to achieve balance between efficiency and robustness. To solve the forward problem, the package uses finite volume method. In this work, we study the use of optimal grid in the package to achieve numerical accuracy to suppress numerical error.

**Keywords:** Resistivity estimation, optimal grid, Finite-Volume inversion, three-dimensional (3-D).

---

### INTRODUCTION

We studied an algorithm for solving the nonlinear electromagnetic inversion problem in the Earth. To achieve a balance between efficiency and robustness, the algorithm employs the Gauss–Newton inversion method. RESINVM3D is an open source 3-D, MATLAB based, resistivity inversion package. This package uses a finite volume discretization to solve the forward partial differential equation.

### RESISTIVITY INVERSION 3D PACKAGE

RESINVM3D includes a highly efficient approach to reducing boundary effects and source electrode singularities. Solving of the forward problem is achieved by using a preconditioned conjugate gradient method. For the inversion algorithm an Incomplete Gauss-Newton solver is used, with the model update being calculated using a preconditioned conjugate gradient algorithm. Default regularization is Tikhonov based on zeroeth and first order derivatives. Moreover, to speed up the inversion's computational time, the so-called optimal grid technique is utilized. The advantage of these optimal grids is that they considerably reduce the computation time without compromising accuracy. Numerical examples for 3-D configurations are used to demonstrate the advantage of using the proposed algorithm over the standard Gauss–Newton inversion method.

### OPTIMAL GRID

Within over the last decades, an approach to computing boundary data for linear second-order problems has been developed which utilizes the concept of “optimal grids” (Druskin, 1997; Ingerman et al, 2000; Druskin &

Knizhnerman, 1999; Asvadurov, Druskin & Knizhnerman, 2000; 2002). This method applies a finite-difference discretization to the second order elliptic operator, using a judiciously chosen sequence of unequal steps to accurately approximate the Neumann-to-Dirichlet (NtD) map associated with that operator in a number of simple geometries. The optimality of the approximation allows to dramatically reduce the number of grid points in the direction normal to the boundary, making the dimensionality of the computational problem essentially equal to that of the boundary. This method has many advantages which make it a natural choice for the numerical studies of nonlinear problems (Muratov & Osipov, 2006; Posta, Shvartsman & Muratov, 2008). In particular, the method is second order-accurate in the size of spatial discretization of the boundary, and the size of the optimal grid can be chosen to match its accuracy with that of the finite-difference stencil on the boundary for all scales of the problem. When very high accuracy of the solution is not required, this approach results in very compact finite-difference approximation schemes for the original PDEs which are typically adequate for computational purposes. Finite Volume method can be considered as a staggered second order finite difference, hence the reasoning for the use of optimal grid in finite difference methods carry over to the finite volume methods used in RESINVM3D.

### NUMERICAL RESULTS

Here we provide a preliminary results of a Numerical example for three-dimensional configurations using the proposed algorithm over the standard Gauss–Newton inversion method.

---

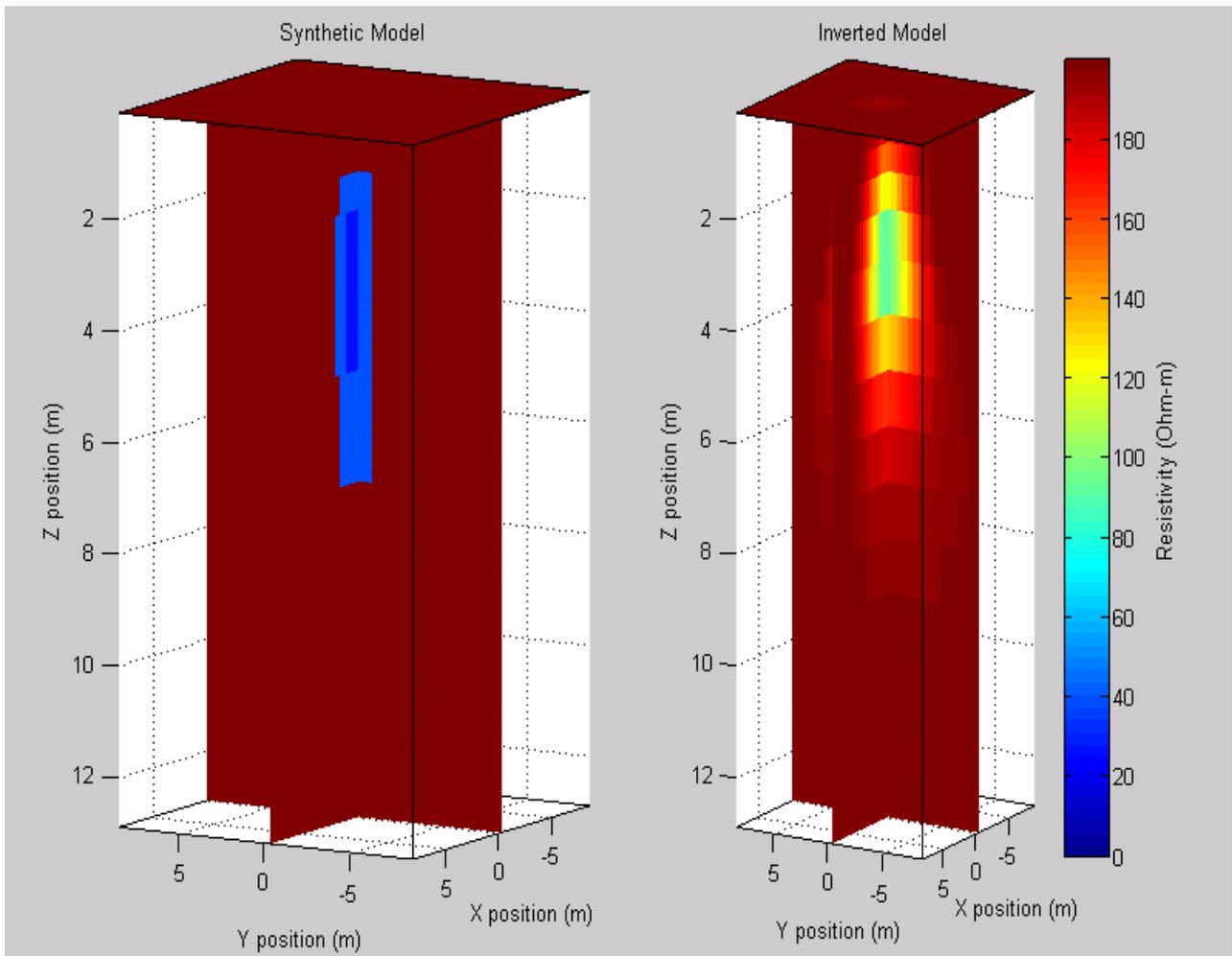


Figure 1. A synthetic case from Pidlisecky, Haber & Rosemary (2007) for benchmarking. On the right hand side is the reconstruction using grids provided in RESINVM3D.

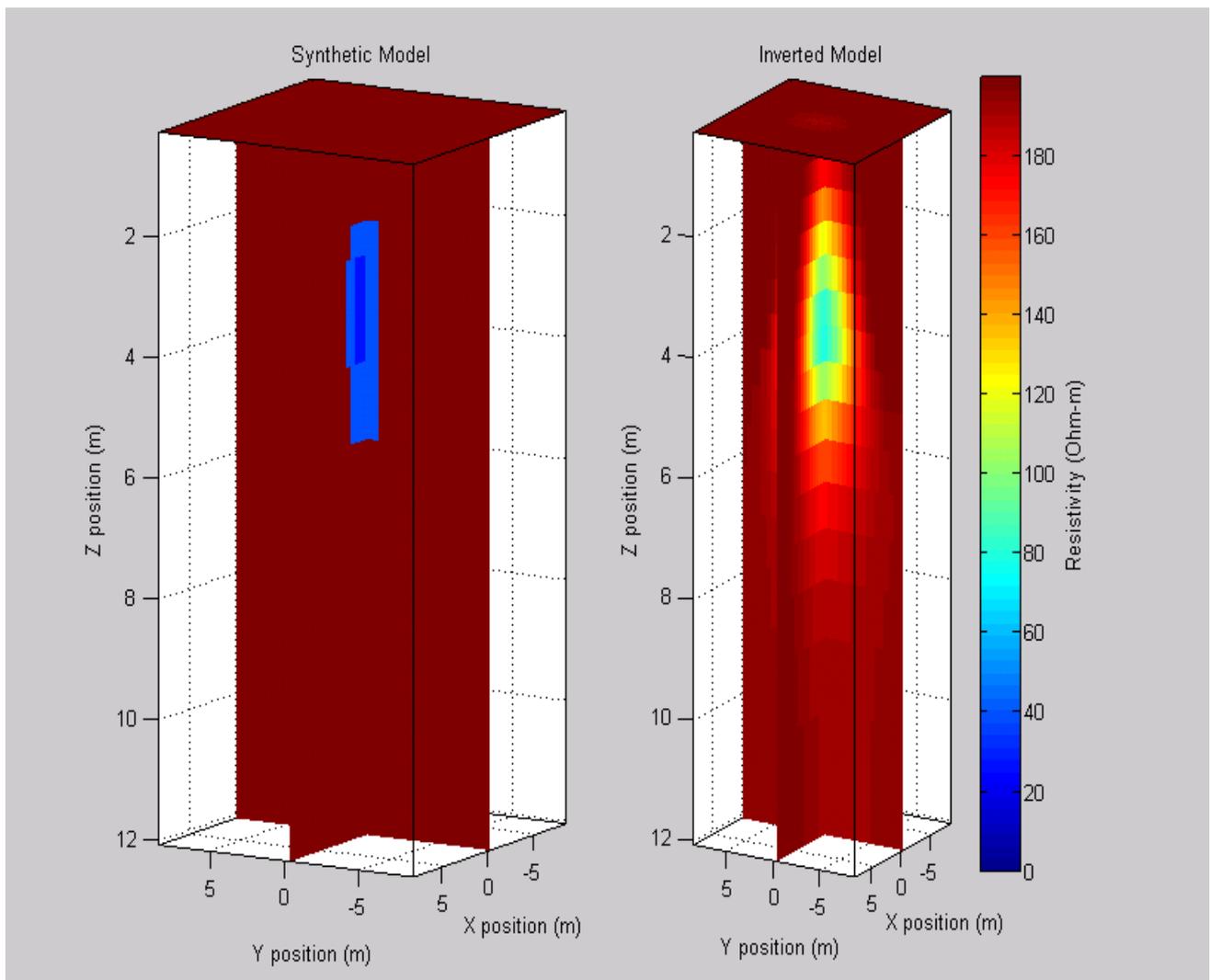


Figure 2. A synthetic case from Pidlisecky, Haber & Rosemary (2007) for benchmarking. On the right hand side is the reconstruction using optimal grids.

## REFERENCES

- Asvadurov, S., Druskin, V. and Knizhnerman, L., 2000, Application of the difference Gaussian rules to solution of hyperbolic problems, *J. Comput. Phys.*, **158**, 116-135.
- Asvadurov, S., Druskin, V. and Knizhnerman, L., 2002, Application of the difference Gaussian rules to solution of hyperbolic problems. II. Global expansion, *J. Comput. Phys.*, **175**, 24-49.
- Druskin, V., Spectrally optimal finite-difference grids in unbounded domains, Schlumberger-Doll Research Notes (1997) EMG-002-97-22, 1997.
- Druskin, V. and Knizhnerman, L., 1999, Gaussian spectral rules for the three-point second differences: I. A two-point positive definite problem in a semiinfinite domain, *SIAM J. Numer. Anal.*, **37**, 403-422.
- Ingerman, D., Druskin, V. and Knizhnerman, L., 2000, Optimal finite-difference grids and rational approximations of the square root: I. Elliptic problems, *Commun. Pure Appl. Math.*, 1039-1066.
- Muratov, C.B. and Osipov, V.V., 2006, Optimal grid-based methods for thin film micromagnetics simulations, *J. Comput. Phys.*, **216**, 637-653.
- Pidlisecky, A., Haber, E. and Knight, R., 2007, RESINVM3D: A 3D resistivity inversion package, *Geophysics*, **72**, H1-H10.
- Posta, F.; Shvartsman, S.Y. and Muratov, C.B., 2008, Compensated optimal grids for elliptic boundary-value problems, *J. Comp. Phys.*, **227**, 8622-8635.