5th International Symposium on Three-Dimensional Electromagnetics
Sapporo, Japan, May 7–9, 2013

# Bootstrapping as a means of uncertainty analysis in inversion modelling of magnetotelluric data

Sebastian Schnaidt[1,2,3] and Graham Heinson[1,2]
[1]Discipline of Geology and Geophysics, The University of Adelaide, Australia
[2]Deep Exploration Technologies Cooperative Research Centre
[3]sebastian.schnaidt@adelaide.edu.au

## SUMMARY

Many geophysical models are created without any form of uncertainty analysis. Mainly because it is not easy to produce a meaningful uncertainty analysis from a single best fit model. Most geophysicists are aware of the limitations of their model, but if the model is passed on to a third party, this information is lost and the risk of misinterpretation arises, which can have serious consequences. We use the bootstrapping resampling method to create reduced data sets from the base data set by random omission of data points. Each of these new data sets is then run through a conventional inversion process to produce an ensemble of solutions with minor variations. The ensemble creation stage is followed by an appraisal stage of statistical analysis of the solution ensembles to infer an uncertainty estimate for the models based on that data set, to increase the reliability of the modelling process. The last step of the workflow is the visualisation and communication of the results to experts as well as non-experts. We demonstrate the effectiveness of the technique with a case study on a magnetotellurics data set from the Southern Delamerian transect in Victoria, Australia. The process yields a clear and easy to interpret uncertainty map for the connected model.

**Keywords:** Bootstrapping, inversion modelling, solution ensemble, uncertainty analysis, magnetotellurics

## INTRODUCTION

These days inversion modelling is widely used to interpret geophysical data. The inversion algorithms become more and more sophisticated and 1D, 2D and 3D data sets are commonly interpreted.
Inversion models are used in a variety of areas, like research and the exploration industry. Data interpretations are a widely traded commodity, which requires the models to be highly reliable to ensure correct interpretation. Even if the person who created the model is aware of its limitations, as soon as the results are handed to a third party, overconfidence in the model bears the risk of misinterpretation.
The problem is that most of today's inversion schemes produce only a single best fit solution and an uncertainty analysis based on a single model has only a limited information value.
We develop methods to create solution ensembles and advanced uncertainty analysis techniques based on these ensembles to increase model reliability. The method we present here is based on bootstrapping resampling.

## METHOD

Bootstrapping is a resampling method used in statistics to calculate sample estimates and was first described by Efron (1979). It is based on statistics calculated from random samples $x^* = (x_1, \ldots, x_m)$, repeatedly drawn from a base data set $X$ ($x^* \subset X$).
We applied this principle to magnetotelluric (MT) data. The behaviour of the electric field $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ involved in MT are governed by diffusion equations (see eq. 1 & 2) (Simpson & Bahr, 2005).

$$\nabla^2 \mathbf{E} = \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} \tag{1}$$

$$\text{and} \quad \nabla^2 \mathbf{B} = \mu_0 \sigma \frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

Because of the diffusive nature of the electromagnetic fields, results from MT measurements represent averages over the volume of medium penetrated by the electromagnetic waves. Thus, data from different sites and different sounding periods $T$ should, depending on the penetration depth $p$ (see eq. 3), contain overlapping information.

$$p = \sqrt{\frac{T\bar{\rho}}{\pi\mu}} \tag{3}$$

The idea of the bootstrapping approach is that for perfect data the random omission of some of the data points should not change the result, as the information is still contained in the remaining data points. Hence, variations in the results give an estimate of the uncertainty inherent in the data.

## PRELIMINARY RESULTS

A first test of the bootstrapping approach was conducted with a data set comprised of broadband MT measurements of 67 sites along the Southern Delamerian transect from Victoria, Australia (Robertson, 2012) (see tab. 1).

**Table 1.** Data set and test specifications.

| | |
|---|---|
| Number of sites | 67 |
| Frequency range | 156.25-0.012 Hz |
| Total number of data points | 8476 |
| Number of bootstrap models | 100 |
| Fraction of data points omitted | 30 % |

A total of 100 inversion models were calculated, based on bootstrapped models with a random data omission of 30 % for each model. All models were based on the same grid and had the same starting model. The inversions were executed with the OCCAM (Constable *et al.*, 1987) 2D smooth inversion code.

Figure 1 shows the result of the standard inversion of the complete data set. The main feature of the model (delineated in black/white) on which we will concentrate here, is a low resistivity structure extending between a depth of 10-40 km and a horizontal position of −10-35 km.

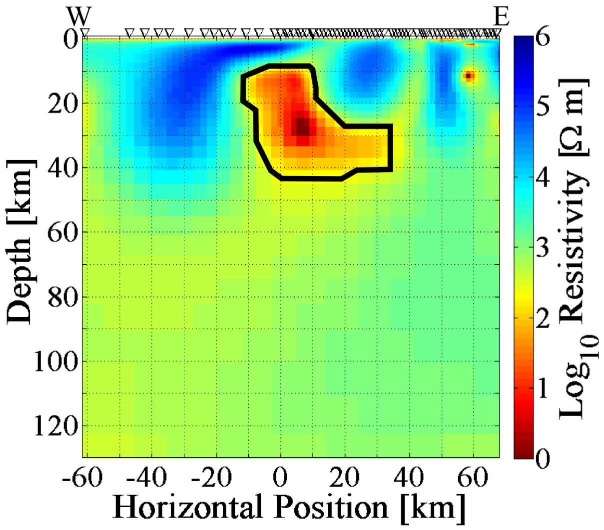


**Figure 1.** Inversion result of the complete data set of the Southern Delamerian MT transect. The shape of the main anomaly is marked in black.

To evaluate the results from the reduced data sets, the standard deviation $s$ (see eq. 4) of the resistivity value $\log_{10}(\rho)$ are calculated for each model cell, under the assumption that the observations $x_i$ are approximately normally distributed (which has not yet been validated).

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad (4)$$

To allow for a comparison of the standard deviation of different cells, the absolute standard deviation is scaled to a relative standard deviation $s_{\text{rel.}} = s/\bar{x}$ (see fig. 2).

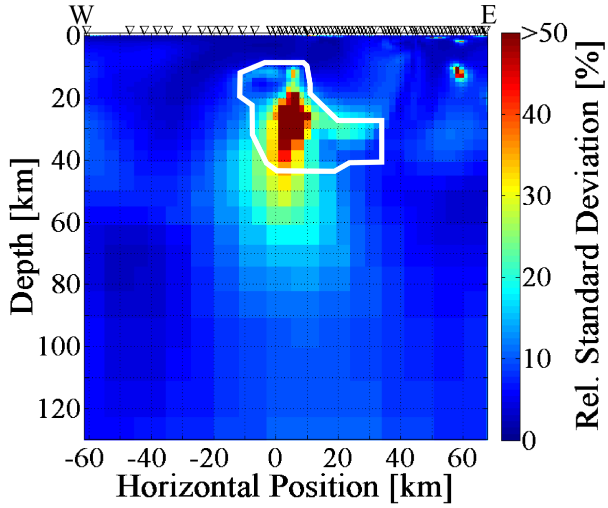


**Figure 2.** Relative standard deviation $s_{\text{rel.}} = s/\bar{x}$ of the resistivity value $\log_{10}(\rho)$ each model cell calculated from 100 inversion models based on 100 reduced data sets (30 % data point omission). Scaled to 0-50 % for higher clarity (max. $s_{\text{rel.}} = 434.59$ %, $\bar{s}_{\text{rel.}}$ = 6.55 %). The shape of the main anomaly, as identified in figure 1, is marked in white.

Figure 2 clearly shows, that the areas of highest uncertainty are strongly correlated to areas of low resistivity.

## DISCUSSION

First results are very promising. As shown in figure 2 the process produces a clear and easy to interpret uncertainty map for the related model (see fig. 1).

As to be expected, the areas of highest uncertainty are connected to the areas of low resistivity, since low resistivity structures cause a strong attenuation of the electromagnetic fields and, hence, lower the achievable resolution. The exact position of those areas differs slightly from the expected though. We expected the areas of high uncertainty to coincide with the lower parts of low resistivity structures, extending to the areas directly underneath the low resistivity structures. That is true for the uprising dome structure in the western part of the anomaly, but in some instances, especially in the eastern half of the main anomaly, the upper edge of the low resistivity structure shows raised uncertainty levels. We are not quite sure what causes this effect, but it might be a shielding effect of the uprising structure to the west. That behaviour needs further examination.

Bootstrapping is superior to the standard sensitivity analysis. A sensitivity analysis only tests the effect of changes

to the model parameters on the model response, which generally just highlights all areas of low resistivity, as those areas are affecting the model response the most. In contrast, bootstrapping directly tests the impact of the data on the model, and flags areas of the model that are poorly constrained.
A disadvantage of the method is that the calculation of the inversion models is very time consuming. That is partially due to the fact that we used an existing standard 2D inversion algorithm and we would probably be able to achieve speed improvements if we would use a dedicated code, but the main factor is just the sheer number of models to be calculated. This absolutely requires the use of multi core machines and is the main reason that this approach is currently only feasible for 2D inversions. The general concept is very much applicable to 3D inversions as well and will yield similarly good results, when rendered possible by the availability of more computing power in the next few years.
Note that, even though we only tested the method on MT data so far, it is generally applicable to all geophysical methods that sample volumes, like e.g. gravity or magnetics.

## OUTLOOK

The project is clearly a work in progress. The next step is to test which type of distribution the observations follow, to confirm that the used statistics are valid or if necessary adjust the statistics accordingly. Furthermore, the effect of different omission percentages and different numbers of bootstrap models will be tested. In addition, we will test the random omission of whole sites instead of random data points and will conduct a second case study with a different high quality data set.

## ACKNOWLEDGMENTS

The authors thank Kate Robertson for providing us with the necessary data, and Lars Krieger for his technical advise. Furthermore, we thank the Deep Exploration Cooperative Research Centre (DET CRC) and its participants for supporting this project.

## REFERENCES

Constable, S. C., Parker, R. L., & Constable, C. G. (1987). Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics*, *52, Nr. 3*, 289-300.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics*, *7, Nr. 1*, 1-26.

Robertson, K. E. (2012). *An electrical resistivity model of the southeast australian lithosphere and asthenosphere*. Honours thesis, The University of Adelaide.

Simpson, F., & Bahr, K. (2005). *Practical magnetotellurics*. United Kingdom: Cambridge University Press.