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Multi-order Vector Finite Element Modelling of 3D Magnetotelluric Data including complex geometry and anisotropic earth

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SUMMARY

We are presenting the progress made on the development of a computational algorithm to model 3D Magnetotelluric data using Vector Finite Element Method (VFEM). The differential equations to be solved are the decoupled Helmholtz equations for the electric field, or the magnetic field, with a symmetric and real conductivity tensor. These equations are defined to include anisotropic earth and complex geometry (such as surface topography, and subsurface interfaces). These governing equations are formulated for the secondary field, where the primary field is the solution of an air domain, homogeneous half-space or layered earth.

This study will compare the effectiveness of two boundary conditions, the Generalize Perfect Matched Layers method (Fang, 1996) versus Dirichlet boundaries. Dirichlet boundary conditions are applied on the tangential fields, assuming that the boundaries lie far away from the inhomogeneities. The Perfect Matched Layer scheme defines an artificial boundary zone that absorbs the propagating electromagnetic fields, to remove the boundary effects. This project will also study the application of a surface boundary in order to remove the air domain, and reduce the model dimension.

In this algorithm, high order edge elements are defined based on covariant projections (Crowley, Silvester, & Hurwitz Jr., 1988) for hexahedral elements. Therefore, vector basis functions are defined for the 12 edges (8 nodes) element, 24 edges (20 nodes) element, and 48 edges (26 nodes) element. By this definition, this vector basis will have zero divergence in the case of rectangular elements and relatively small divergence in the case of distorted elements. They are defined to study their numerical accuracy and speed, and see if the divergence correction is automatically satisfied.

Keywords: 3D forward modeling, vector finite element, MT method, anisotropy

INTRODUCTION

EM modelling numerically solves the Maxwell's partial differential equations (PDEs), with a source field and boundary conditions. In geophysical terms, this forward model can be seen as the generation of synthetic EM data from a specific geo-EM model of conductivity (σ), and magnetic permeability (μ). In the case of MT modelling, these PDEs are solved with a propagating plane wave as a source field, and taking into consideration the quasi-static condition. This procedure is an important step for the inversion of MT data, which obtains successive geo-EM models that yields to synthetic MT data fitting the measured one. Therefore, the inversion needs a fast, accurate and reliable MT modeling solution (Avdeev, 2005; Börner, 2010).

Sometimes in MT applications, the measured data can't be properly inverted with 1D or 2D models, so it is necessary to obtain reliable 3D models to could interpret these data. The main numerical techniques applied to 3D MT modelling are Finite Differences (Mackie, Madden, & Wannamaker, 1993; Weiss & Newman, 2002, 2003; Haber & Heldmann, 2007), Integral Equations (Wannamaker, 1991; Zhdanov, Lee, & Yoshioka, 2006) and Finite Element (Farquharson & Miensopust, 2011; Mitsuhata & Uchida, 2004; Nam et al., 2007; Shi, Utada, Wang, & Wu, 2004) methods. The main problem of 3D EM modelling is the computational memory and time required to obtain the solutions. This has been overcomed by applying a parallel framework in the modelling technique. Therefore, the main goal of these numerical implementations is to reduce the computational load and obtain accurate solutions including anisotropy and complex model geometry (Avdeev, 2005).

In this project, a Vector Finite Element Method (VFEM) is being developed with multi-order vector basis functions. The basis functions are based on covariant projections for hexahedral elements (Crowley et al., 1988). Vector basis functions are defined for linear (12 edges, 8 nodes), quadratic (24 edges, 20 nodes), and Lagrangian (48 edges, 26 nodes) hexahedral elements. The effectiveness of Dirichlet boundary conditions, and Generalize Perfect Matched Layers method (Fang, 1996) will be compared in this study. An analysis of convergence of this model to analytical solutions (homogeneous or layered earth) will be carried out, in order to validate the program. Also, information of sampling densities, extension zones and air layer height that yields to faster and accurate models will be obtained.

MT PROBLEM

The governing equations to be solved are the decoupled Helmholtz equations for the electric field ($\vec{\mathrm{E}}$), and magnetic field ($\vec{\mathrm{H}}$). For MT modelling, these equations (1) are obtained with the quasi-static conditions, and in terms of the secondary field formulation (e.g. $\vec{\mathrm{E}} = \vec{\mathrm{E}}^s + \vec{\mathrm{E}}^p$). The primary field $\vec{\mathrm{E}}^p$ is the solution of a plane wave propagating on either the air domain, a homogeneous earth, or a layered earth. It should be notice that this formulation takes into consideration the anisotropy, by defining $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ as symmetric tensors.

$$\begin{aligned}
&\nabla \times (\boldsymbol{\mu}^{-1} \cdot \nabla \times \vec{\mathrm{E}}^s) + i\omega\boldsymbol{\sigma} \cdot \vec{\mathrm{E}}^s = \vec{\mathrm{s}}_E \\
&\vec{\mathrm{s}}_E = -i\omega[\delta\boldsymbol{\sigma} \cdot \vec{\mathrm{E}}^p + \nabla \times (\boldsymbol{\mu}^{-1}\delta\boldsymbol{\mu} \cdot \vec{\mathrm{H}}^p)] \\
&\nabla \times (\boldsymbol{\sigma}^{-1} \cdot \nabla \times \vec{\mathrm{H}}^s) + i\omega\boldsymbol{\mu} \cdot \vec{\mathrm{H}}^s = \vec{\mathrm{s}}_H \\
&\vec{\mathrm{s}}_H = -i\omega\delta\boldsymbol{\mu} \cdot \vec{\mathrm{H}}^p + \nabla \times (\boldsymbol{\sigma}^{-1}\delta\boldsymbol{\sigma} \cdot \vec{\mathrm{E}}^p)]
\end{aligned} \tag{1}$$

In eq.(1), $\delta\boldsymbol{\sigma} = \boldsymbol{\sigma}^s - \boldsymbol{\sigma}^p$ is the difference between the geo-EM model of the primary fields and the geo-EM model of the unknown secondary field. Usually, one of these equations is solved and the other field is obtained directly from the numerical application of Maxwell's equations. These equations are solved for the TE and TM modes, to obtain the impedance of the subsurface, and be able to be applied into some inversion routine.

VECTOR FINITE ELEMENT METHOD

Applying the Galerking method to the governing equations (1), is possible to obtain the integrals to be numerically solved with a vector function $\vec{\mathrm{w}}$ (2).

$$\begin{aligned}
&\int_\Omega \begin{pmatrix} (\nabla \times \vec{\mathrm{w}}) \cdot \boldsymbol{\mu}^{-1} \cdot (\nabla \times \vec{\mathrm{E}}^s) + i\omega\vec{\mathrm{w}} \cdot \boldsymbol{\sigma} \cdot \vec{\mathrm{E}}^s \\ (\nabla \times \vec{\mathrm{w}}) \cdot \boldsymbol{\sigma}^{-1} \cdot (\nabla \times \vec{\mathrm{H}}^s) + i\omega\vec{\mathrm{w}} \cdot \boldsymbol{\mu} \cdot \vec{\mathrm{H}}^s \end{pmatrix} d\Omega \\
&= c_\Gamma \int_\Gamma \begin{pmatrix} \vec{\mathrm{w}} \cdot (\hat{\mathrm{n}} \times \vec{\mathrm{H}}_0^s) \\ \vec{\mathrm{w}} \cdot (\vec{\mathrm{E}}_0^s \times \hat{\mathrm{n}}) \end{pmatrix} d\Gamma + \int_\Omega \begin{pmatrix} \vec{\mathrm{w}} \cdot \vec{\mathrm{s}}_E \\ \vec{\mathrm{w}} \cdot \vec{\mathrm{s}}_H \end{pmatrix} d\Omega
\end{aligned} \tag{2}$$

The model domain Ω, including topography and subsurface interfaces Fig. 1, is discretized with hexahedral elements, and extension zones Fig. 2. Within each hexahedral element, the unknown field $\vec{\mathrm{F}}^{(e)}$ can be obtained from a vector basis function (3), where ME is the number of edges in the element. The vector basis function, eq. (4), is defined in terms of the nodal basis function of global coordinates $\mathrm{N}_\alpha^{(e)}(\vec{\mathrm{r}})$, and a normalized edge vector $\vec{w}_\alpha$. This edge vector is obtained from the covariant projections $\vec{\xi}$ in local coordinates (Crowley et al., 1988). In order to obtain edge vectors that can be shared with other adjacent elements, the contribution of the nodes i and j that defines the edge must be taken into account. Using the numerical field eq. (3) in eq. (2), for $\vec{\mathrm{w}}$ and $\vec{\mathrm{E}}^s$ or $\vec{\mathrm{H}}^s$, then the integrals can be numerically solved by Gauss quadrature. This numerical application yields to a sparse system of equations, which can be stored as non-zero entries. In this algorithm, the MUMPS package is used to solve the system of equations.

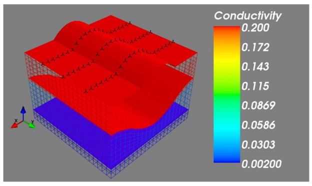

Figure 1: Inner model with topography and interfaces.

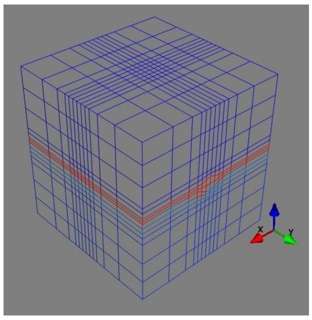

Figure 2: Domain discretization with extension zones.

In MT modelling, 3D VFEM has been applied with vector basis function of linear order on a rectilinear mesh (Farquharson & Miensopust, 2011; Mitsuhata & Uchida, 2004; Shi et al., 2004) or hexahedral elements (Nam et al., 2007). In this methodology, three different order of basis functions has been developed. These are, Linear (8 nodes, 12 edges Fig. 3), Quadratic (20 nodes, 24 edges, Fig. 4), and Lagrangian (26 nodes, 48 edges, Fig. 5) basis functions. The accuracy, time and computational load required for these basis function cases will be studied.

$$\vec{\mathrm{F}}^{(e)} = \sum_{ME}^{\alpha=1} F_\alpha^{(e)} \vec{\mathrm{V}}_\alpha^{(e)}(\vec{\mathrm{r}}) \tag{3}$$

$$\vec{\mathrm{V}}_{\alpha}^{(e)}(\vec{\mathrm{r}}) = \mathrm{N}_{\alpha}^{(e)}(\vec{\mathrm{r}})\vec{w}_{\alpha}$$
$$\mathrm{N}_{\alpha}^{(e)}(\vec{\mathrm{r}}) = \sum_{l=0}^{2}\sum_{m=0}^{2}\sum_{n=0}^{2} c_{lmn}x^{l}y^{m}z^{n} \qquad (4)$$
$$\vec{w}_{\alpha} = \frac{\vec{\xi}_i + \vec{\xi}_j}{\left|\vec{\xi}_i + \vec{\xi}_j\right|}; \vec{\xi}_i = \frac{\partial \vec{\mathrm{r}}}{\partial \xi}$$

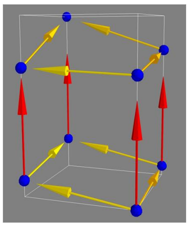

Figure 3: Linear edges

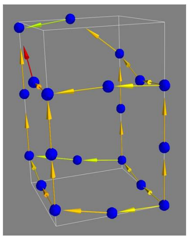

Figure 4: Quadratic Edges

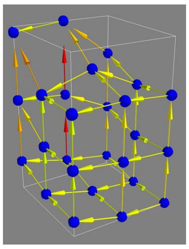

Figure 5: Lagrangian Edges

BOUNDARY CONDITIONS

The most common boundary condition in MT applications is Dirchlet boundary. This condition considers that the boundaries lies far away from the inhomogeneities, so the fields vanish at the boundary by assigning a value of zero to the tangential fields (e.g. $\hat{\mathrm{n}} \times \vec{\mathrm{H}}_0^s = 0$). In this case, we don't need to consider the boundary integration in eq. (2), so $c_\Gamma = 0$. The problem with this boundary condition is that sometimes is not possible (computationally) to extend the domain until the condition is achieved. This brings some field reflections from the boundaries back to the inhomogeneities, making the solution unaccurate.

To deal with this problem, Berenger (1994) proposed an absorbing boundary zone where the fields decay with distance until it vanish at the boundary of the domain. This Perfect Matched Layer (PML) absorbs the waves that strikes it, without reflecting it backwards. This is done by introducing a complex coordinate stretching factor to the governing equations. In this project, both Dirichlet and a Generalized PML (Fang, 1996) will be applied in order to compare the effectiveness of both conditions.

Another boundary condition of interest in this study, is the surface boundary. A modification of the surface boundary integrals has been made (Zhou, Heinson, & Rivera-Rios, 2012), taking in consideration the anisotropy of the subsurface. With this anisotropic boundary condition we may calculate the fields on the surface from the underground fields. Therefore, we may exclude the air domain from the computation of the unknown field (Zhou et al., 2012).

OUTCOMES

The main outcome of this project will be a 3D MT forward modelling technique in frequency domain, for isotropic and anisotropic media, with complex topography and subsurface interfaces. A multi-order VFEM based on covariant projection edges is being developed. This method will be validated by an analysis of convergence of the model with analytical data. The analytical data considered are wave propagation in the air, in a homogeneous half-space, and in layered earth. With this analysis, the sampling density, and air layer height that yields to the faster, and accurate models will be obtained. A comparison of the different basis function orders will be carried out. As well as a comparison of the different boundary condition applications.

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